An Algebraic Approach for Path-Diversity Multiple-Description-Coding Scenario

Pierluigi Salvo Rossi [1]
Dipartimento di Informatica e Sistemistica
Università di Napoli "Federico II"
Via Claudio 21
80125 Napoli, Italy
e-mail: `salvoros@unina.it`

Francesco Palmieri
Dipartimento di Ingegneria dell'Informazione
Seconda Università di Napoli
Real Casa dell'Annunziata
Via Roma 29
81031 Aversa (CE), Italy,
e-mail: `frapalmi@unina.it`

Giulio Iannello
Università Campus Bio-Medico di Roma
Via Longoni 83
00155 Roma, Italy
e-mail: `iannello@unina.it`

Abstract — **Packet transmission over computer networks is the main reference framework for most information technology applications. Packet loss is one of the main problems affecting performance. Various techniques have been proposed to protect packets from losses, one is distributing information on the network in order to exploit the characteristics of the network itself allowing adaptivity and flexibility. In this work we propose an algebraic framework for estimate of the number of independent paths (Path Diversity) on a network. Path Diversity knowledge is showed to be extremely important for applications making use of distributed information. Joint use of Path Diversity and Multiple Description Coding allows more efficient transmissions and can be generalized to the case when network coding is allowed on internal nodes of the network.**

I. Introduction

In the current information society, most communication travels through an integrated network of heterogeneous links (the Internet). Packet transmission is the main reference framework for most applications that are based on very different kinds of connection devices, type of users and applications.

The main phenomenon affecting packet-switched networks is packet loss. It can be due to buffer overflow in internal nodes, to packet discarding in the presence of error, to timeout for excessive delays, etc. Several strategies to overcome this limitation have been proposed in the literature, such as Forward Error Correction based on redundant codes, or robust transforms [2][6][8]. Multiple Description Coding (MDC) is usually related to the availability of redundant paths connecting the source to the sink. Multicast transmission is also based on the availability of multiple ways to connect sources and sinks. Exploiting multiple paths in a packet-switched network is a way to make more robust the transmission, and the effectiveness depends on the correlation among the alternative paths [4]. Effective packet routing in combination with MDC can be very useful in the new communication paradigm [5][7].

[1]This work has been carried out partially under the financial support of the Ministero dell'Istruzione, dell'Università e della Ricerca (MIUR) in the framework of the FIRB Project "Middleware for advanced services over large-scale, wired-wireless distributed systems (WEB-MINDS)".

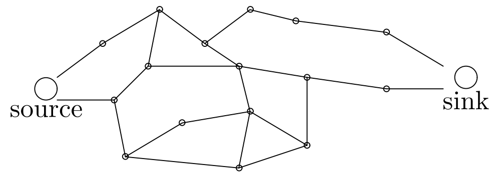


Figure 1: Network.

The large variety of techniques for packet transmission makes very difficult a unique analysis for jointly designing source coding and transmission strategies on real packet networks. In this work we propose an algebraic framework that allows a compact analysis of various network topologies built out of source nodes, routing nodes, coding nodes, links, etc. Given a source-sink pair connected via a packet network, as shown in Fig. 1, we are interested in obtaining an equivalent representation of the connection, that can be manipulated to evaluate MDC and path diversity performance, for efficient transmission design.

In this paper we refer our analysis only to a generic source, without taking into account the time evolution of both source and sink. The physical links of the network are assumed to be *iid* systems, neglecting the effect of delay as well as the possible bursty behavior of the random losses. The transmission of the single packet on the network will be analyzed.

II. The Z-Channel

The simplest configuration, a single link that connects one source to one sink, is shown in Fig. 2. The link is assumed to be lossy, i.e. the packet at the source is not delivered with certainty at the sink, but it may be lost with a given probability. We do not take into account the specific information associated to the packet, but only refer to presence or absence of the packet itself.

A link between a source and a sink can be described via the Z-channel shown in Fig. 3, where 0 and 1 denotes absence and presence of a packet. The Z-channel describes the evidence that:

- if there is no packet at the source there will be no packet at the sink;
- if there is a packet at the source the packet will be successfully transmitted at the sink with probability $1-$

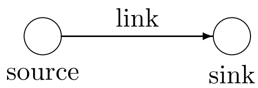


Figure 2: Single link.

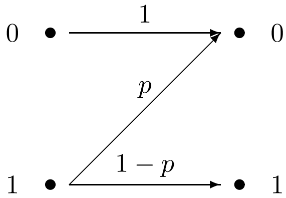


Figure 3: Z-channel.

p, otherwise there will be no packet at the sink (with probability p).

The Z-channel is obviously associated to the matrix

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ p & 1-p \end{pmatrix} . \quad (1)$$

II.A Basic Configurations

Fig. 4 shows the equivalent Z-channel for some typical configurations such as *serial connection* and *parallel connection*.

The serial connection of 2 links between a source and a sink (assuming that both $link_1$ and $link_2$ are described via Eq. (1)) can be described via the equivalent Z-channel

$$\begin{aligned} \mathbf{P}_{\text{serial}} &= \mathbf{P} \cdot \mathbf{I} \cdot \mathbf{P} = \mathbf{P}^2 \\ &= \begin{pmatrix} 1 & 0 \\ p(2-p) & (1-p)^2 \end{pmatrix} , \end{aligned}$$

where we assume that the intermediate node behaves like the identity operator. The parallel connection of 2 links between a source and a sink (assuming again that both $link_1$ and $link_2$ are described via Eq. (1)) can be described via the equivalent Z-channel

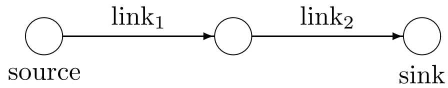
$$\begin{aligned} \mathbf{P}_{\text{parallel}} &= \mathbf{D}_2 \cdot (\mathbf{P} \otimes \mathbf{P}) \cdot \mathbf{C}_2 \\ &= \begin{pmatrix} 1 & 0 \\ p^2 & 1-p^2 \end{pmatrix} , \end{aligned} \quad (2)$$

where $\otimes$ denotes the Kronecker product, and

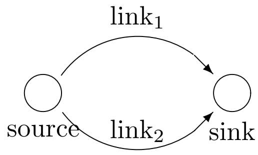
$$\mathbf{D}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \quad \mathbf{C}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} ,$$

are the matrices associated to a diverging and a converging nodes respectively. Eq. (2) is easily interpreted when noted that

- $link_1$ and $link_2$ jointly represent a quaternary-input quaternary-output block associated to $\mathbf{P} \otimes \mathbf{P}$ (a 4×4 matrix);



(a) Serial connection.



(b) Parallel connection.

Figure 4: Basic connections.

- the source is a *diverging node*, a binary-input quaternary-output block, as shown in Fig. 5, associated to D_2 (a 2×4 matrix);
- the sink is a *converging node*, a quaternary-input binary-output block, as shown in Fig. 5, associated to C_2 (a 4×2 matrix).

The generalization is

$$\begin{aligned} \mathbf{P}_{\text{serial}} &= \prod_{i=1}^{N} \mathbf{P}_i , & (3) \\ \mathbf{P}_{\text{parallel}} &= \mathbf{D}_N \cdot (\otimes_{i=1}^{N} \mathbf{P}_i) \cdot \mathbf{C}_N , & (4) \end{aligned}$$

for serial and parallel connections with N links, being $\mathbf{D}_N$ and $\mathbf{C}_N$ a 2×2^N matrix and a $2^N \times 2$ matrix respectively, whose elements are

$$\begin{aligned} \mathbf{D}_N(i,j) &= \begin{cases} 1 & {\scriptstyle (i,j)=(1,1),(2,2^N)} \\ 0 & \text{otherwise} \end{cases} , \\ \mathbf{C}_N(i,j) &= \begin{cases} 1 & {\scriptstyle (i,j)=(1,1),(2,2),(3,2),\ldots(2^N,2)} \\ 0 & \text{otherwise} \end{cases} . \end{aligned}$$

Denoting p_{eq} the global loss probability of the connection, from Eqs. (3) and (4) for serial and parallel connections with N identical links we have

$$\begin{aligned} \mathbf{P}_{\text{serial}} &= \begin{pmatrix} 1 & 0 \\ 1-(1-p)^N & (1-p)^N \end{pmatrix} , \\ \mathbf{P}_{\text{parallel}} &= \begin{pmatrix} 1 & 0 \\ p^N & 1-p^N \end{pmatrix} , \end{aligned}$$

that is

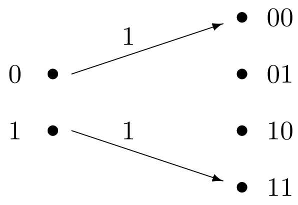
$$p_{eq} = 1 - (1-p)^N ,$$

for serial connection, and

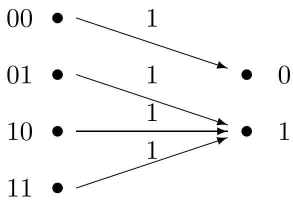
$$p_{eq} = p^N ,$$

for parallel connection.

This means, as shown in Fig. 6, that



(a) Diverging node.



(b) Converging node.

Figure 5: Converging and diverging nodes.

- on a serial connection increasing the number of links does not change the slope of the p_{eq}-p curve;
- on a parallel connection the slope of the p_{eq}-p curve increases with the number of links.

Therefore the slope is associated to the parallelism of the whole connection between the source and the sink when reduced to an equivalent simple configuration.

III. Z-channel and Path Diversity

A general network topology can be represented as a Z-channel via

$$\mathbf{P}_{eq} = \begin{pmatrix} 1 & 0 \\ p_{eq} & 1 - p_{eq} \end{pmatrix} ,$$

by using Eqs. (3) and (4), being the parameter p_{eq} the global loss probability of the network between the source ad the sink.

Furthermore, the parameter p_{eq} of the equivalent Z-channel has a polynomial expression with respect to p (assuming that the network is composed by equal links),

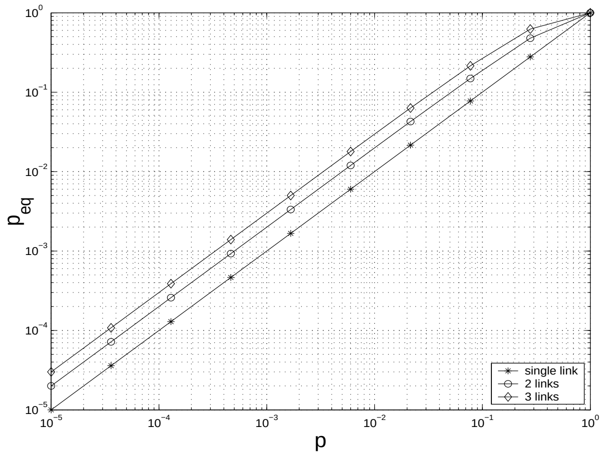
$$p_{eq} = \sum_{k=1}^{N} C_k \; p^k , \qquad (5)$$

then the slope of the p_{eq}-p curve is given by the minimum power in the polynomial expression of Eq. (5). According to this, it is useful defining the *Path Diversity* (L) of a network as

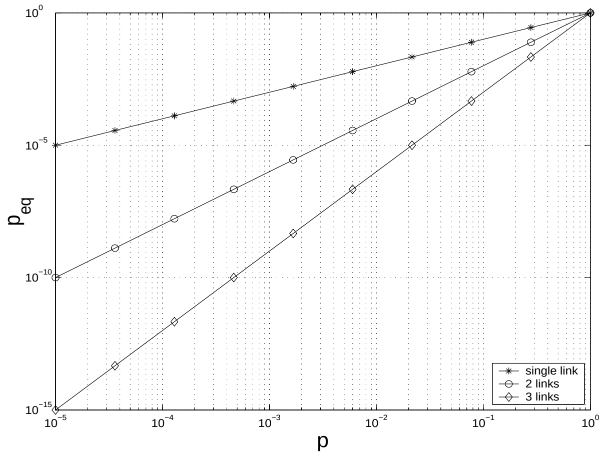
$$L = \min_{C_k \neq 0} k , \qquad (6)$$

being C_k the coefficient of the k-th power of the link loss probability (p) in the polynomial expression of the equivalent Z-channel loss probability (p_{eq}).

On the basis of considerations in Section II.A, the Path Diversity of a network represents the number of parallel independent path connecting the source to the sink.



(a) Serial connection.



(b) Parallel connection.

Figure 6: Dependence of p_{eq} with respect to number of links for basic connections.

Fig. 7 shows 2 examples of network topologies that may emerge from a real network in which we are looking for 2 independent paths. It is easy to compute the equivalent Z-channel for both of them, obtaining

$$\begin{aligned} \mathbf{P}_{eq,A} &= \mathbf{D} \cdot (\mathbf{P}^2 \otimes \mathbf{P}^2) \cdot \mathbf{C} \\ &= \begin{pmatrix} 1 & 0 \\ p^2(2-p)^2 & 1 - p^2(2-p)^2 \end{pmatrix} , \\ \mathbf{P}_{eq,B} &= \mathbf{D} \cdot (\mathbf{P} \otimes \mathbf{P}) \cdot \mathbf{C} \cdot \mathbf{P} \\ &= \begin{pmatrix} 1 & 0 \\ p(1+p-p^2) & 1 - p(1+p-p^2) \end{pmatrix} . \end{aligned}$$

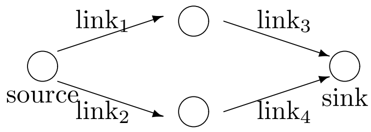
that is

$$\begin{aligned} p_{eq,A} &= p^2(2-p)^2 , \\ p_{eq,B} &= p(1+p-p^2) . \end{aligned}$$

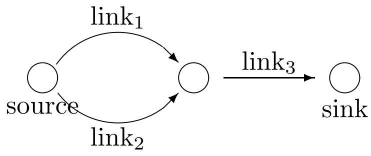
Fig. 8 compares the p_{eq}-p curves with basic configurations confirming that $L_A = 2$ and $L_B = 1$.

IV. Joint Use of Path Diversity and Multiple Description Coding

Object of this work is to show how Path Diversity can be used for efficient transmission schemes on computer networks.



(a) NT-A.



(b) NT-B.

Figure 7: Network topology examples.

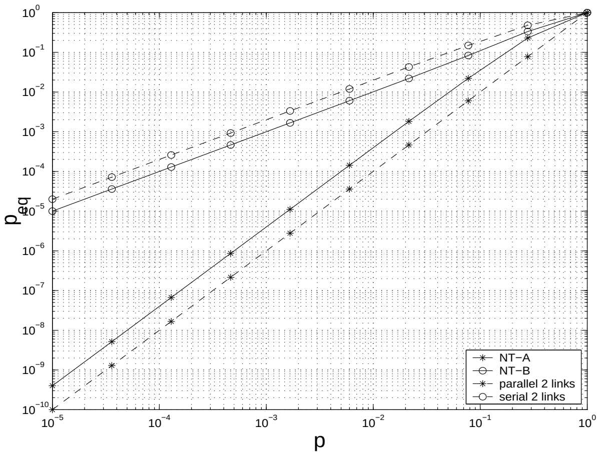


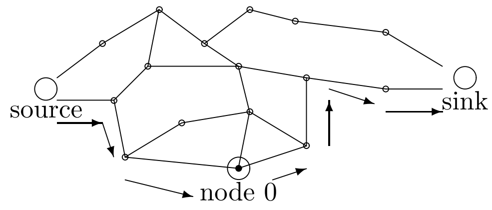
Figure 8: Comparing network topology with serial and parallel connections.

The framework allows joint manipulation of the spatial diversity offered by the network [4] and the coding diversity offered by encoding schemes such as MDC [2][6]. It will be shown how distributing information over the network improves the overall performance if the network topology can support it, otherwise it can lead to significant degradation.

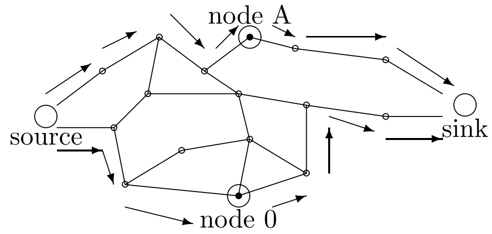
IV.A Coding Assumptions

We suppose that source is allowed to encode each packet via MDC with an arbitrary number of descriptions. We assume that the encoding scheme is characterized by a quality function describing the Quality of Service (QoS) at the sink, depending on the amount of bits (i.e. the number of descriptions) that have been delivered. We also assume that the quality function ($q(.)$) only depends on the fraction of delivered bits (y), and such that

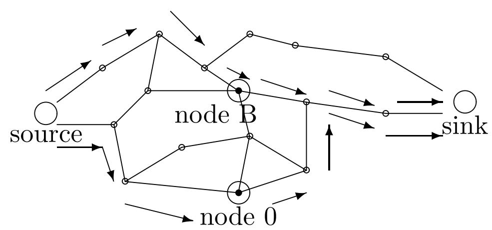
$$\begin{cases} q(0)=0 \quad , \quad q(1)=1 \\ \frac{d}{dy}q(y) \geq 0 \\ \frac{d^2}{dy^2}q(y) \leq 0 \end{cases} .$$



(a) Single path. The connection is equivalent to a serial connection, i.e. $L = 1$.



(b) 2 different paths. The connection is equivalent to NT-A, i.e. $L = 2$.



(c) 2 different paths. The connection is equivalent to NT-B, i.e. $L = 1$.

Figure 9: Selecting relay-nodes on the network.

The QoS at the sink is intended as the average value of the quality function,

$$Q = E\{q(y)\} = \sum_i \pi_i q(y_i) \ , \tag{7}$$

where y_i represents the fraction of delivered bits in the i-th loss configuration of the network, and π_i is the probability that the i-th loss configuration occurs. With the term *loss configuration* we mean the event that only the fraction of bits y_i, with respect to the whole bits transmitted by the source, are correctly delivered to the sink.

IV.B Network Assumptions

We assume that the characteristics of the links composing the network are known. According to this, the sink is able to determine the order of diversity of the network between the source and the sink itself. This can be done via:

- estimation of the parameter p_{eq} by observation of delivered packets;
- individuation of the point of operation in the p_{eq}-p plane.

Fig. 9 shows the same network shown in Fig. 1 in 3 different situations depending on the way the source is sending its packets to the sink. More specifically:

- Fig. 9(a) shows the case in which the source transmits packets to the sink via a single path through *node* 0. The whole system can be modeled as a serial connection.
- Fig. 9(b) shows the case in which the source transmits packets to the sink via 2 different paths through *node* 0 and through *node* A. The whole system can be modeled as the network topology NT-A in Fig 7(a).
- Fig. 9(c) shows the case in which the source transmits packets to the sink via 2 different paths through *node* 0 and through *node* B. The whole system can be modeled as the network topology NT-B in Fig 7(b).

We suppose that the sink is able to recover that the order of diversity for the 3 cases are $L_1 = 1$, $L_A = 2$, $L_B = 1$, respectively.

IV.C Exploiting Path Diversity

We are interested in showing how Path Diversity knowledge, i.e. individuating the number of independent paths connecting the source to the sink, suggests the appropriate number of descriptions in MDC schemes. Moreover, on the basis of the examples shown in Section IV.B we want to show how the use of MDC without appropriate knowledge of Path Diversity may turn into degradation of overall performance instead of making more robust the transmission scheme.

In order to evaluate the performance in the 3 situations shown in Fig. 9, we compute the performance on a serial connection, shown in Fig. 4(a), and on network topologies NT-A and NT-B, shown in Figs. 7(a) and 7(b), respectively. We refer our analysis to a generic source whose packet are encoded according to the criteria described in Section IV.A, and furthermore we restrict our attention to the single packet transmission ignoring the time evolution of both source and receiver.

Consider the case in which the packet is encoded via a single description and simply transmitted to the sink. In this case the possible loss configurations are:

1. description is delivered, corresponding to $q(1)$;
2. description is not delivered, corresponding to $q(0)$.

It is clear that the performance of the serial connection, representing the situation shown in Fig. 9(a), are

$$Q_1 = (1-p)^2 , \tag{8}$$

as we are assuming that $q(0) = 0$ and $q(1) = 1$.

Now consider that the source encodes the packet via 2 descriptions with equal amount of bits and encoded information. In this case the possible loss configurations are:

1. both descriptions are delivered, corresponding to $q(1)$;
2. first description is delivered and second one is not, corresponding to $q(1/2)$;
3. second description is delivered and first one is not, corresponding to $q(1/2)$;
4. no description is delivered, corresponding to $q(0)$.

A more efficient utilization of this encoding scheme is obtained if 2 different paths are found.

The performance of NT-A, representing the situation shown in Fig. 9(b), are

$$Q_A = (1-p)^2((1-p)^2 + 2p(2-p)q(1/2)) . \tag{9}$$

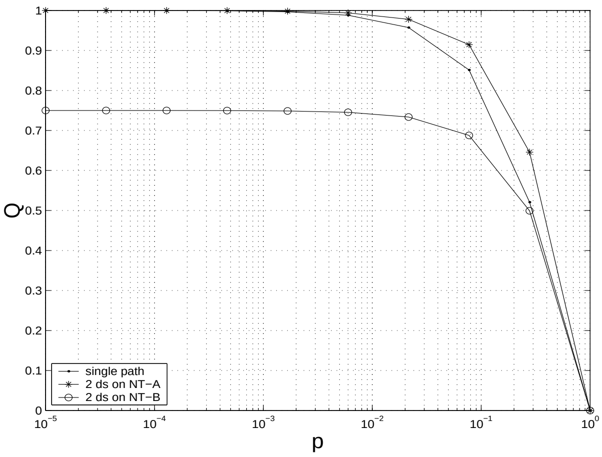
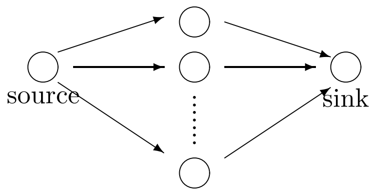
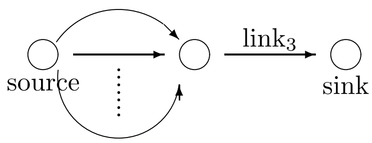


Figure 10: MDC and Path Diversity, $q(1/2) = 0.75$.



(a) NT-A.



(b) NT-B.

Figure 11: Network topology examples.

The performance of NT-B, representing the situation shown in Fig. 9(c), are

$$Q_B = (1-p^2)(1-p)q(1/2) , \tag{10}$$

Fig. 11 shows the generalization to N paths, whose performance, in the case that N descriptions are used, can be shown to be

$$\begin{aligned} Q_{A,N} &= \sum_{n=1}^{N} \left(\begin{array}{c} n \\ N \end{array} \right) p^{N-n}(2-p)^{N-n}(1-p)^{2n} q\left(\frac{n}{N}\right) , \\ Q_{B,N} &= (1-p)(1-p^N)q\left(\frac{1}{N}\right) . \end{aligned}$$

In Fig. 10 the performance of the 3 situations Q_1, Q_A, Q_B are plotted (the dot-, asterisk-, circle-solid lines respectively) with respect to the loss probability (p) of the link. It can be noted how changing from a 1-description encoding scheme with a single path to a 2-description one with 2 different paths like NT-A improves the QoS at the receiver. On the contrary, changing to a 2-description encoding scheme with 2 different paths like NT-B noticeably degrades the performance (25%)

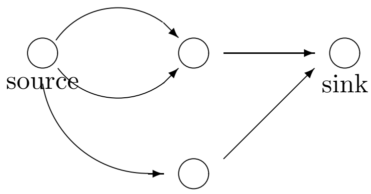


Figure 12: Network topology example.

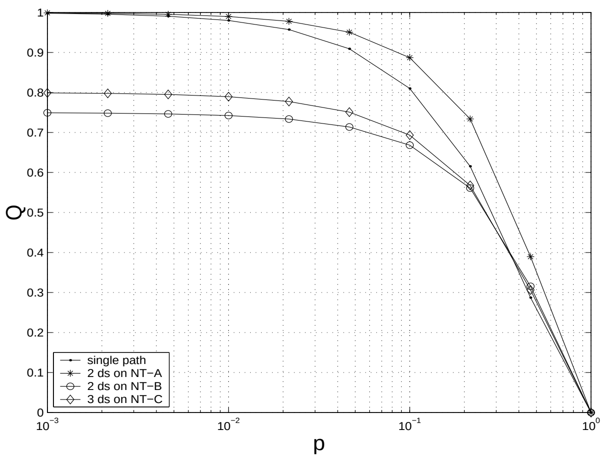


Figure 13: MDC and Path Diversity, $q(1/2) = 0.75$, $q(1/3) = 0.45$, $q(2/3) = 0.80$.

as the network topology (having order of diversity $L_B = 1$) is not able to efficiently support 2 descriptions on 2 independent paths.

Fig. 12 shows another example of network topology whose equivalent Z-channel is

$$\mathbf{P}_{eq,C} = \mathbf{D}_2 \cdot ((\mathbf{D}_2 \cdot (\mathbf{P} \otimes \mathbf{P}) \cdot \mathbf{C}_2 \cdot \mathbf{P}) \otimes \mathbf{P}^2) \cdot \mathbf{C}_2$$
$$= \begin{pmatrix} 1 & 0 \\ p^2(2-p)(1+p-p^2) & 1 - p^2(2-p)(1+p-p^2) \end{pmatrix} .$$

meaning that the order of diversity is $L_C = 2$, while Fig. 13 shows the performance on this network topology for

- 1 description, i.e. Eq. (8);
- 2 descriptions, i.e. Eq. (9) or Eq. (10);
- 3 descriptions, i.e.

$Q_C = p(1-p)^2(3+2p-2p^2)q\left(\frac{1}{3}\right) + (1+p)(1-p)^4 q\left(\frac{2}{3}\right)$ showing how the use of 3 description leads to performance degradation.

This clearly shows how Path Diversity knowledge can be helpful for choosing the appropriate number of descriptions or appropriate redundant paths in an MDC environment. Assume that the sink is able to estimate the order of diversity of the network topology (from the source to itself) and send this information back to the source. Then, on the basis of the received information, the source obtains confirmation of an efficient transmission for situations like NT-A, while alternative paths or reduction of number of descriptions have to be tried for situations like NT-B in order not to waste resources.

Note that we assumed that no network coding [1][3] is allowed at internal nodes. Otherwise, if network coding is allowed and each node is able to combine different received descriptions, then each internal node can act as a receiver, estimate its own order of diversity, send the information back to the source and optimize the global transmission scheme. Referring again to the example NT-B of Fig. 7(b), if network coding is allowed then transmission from source to the internal node may support a 2-description scheme, while the internal node has to combine the received description into a single one before transmitting to the sink.

V. Conclusion and Future Work

In this work we have proposed an algebraic approach to analyze the Path Diversity offered by a computer network. We have also showed how knowledge of Path Diversity is crucial for applications, like Multiple Description Coding schemes, making use of distributed information. Erroneous Path Diversity information can lead to significant degradation in the overall performance instead of the expected robustness improvement. Path Diversity estimation may be used to select the appropriate number of descriptions or to evaluate on the effectiveness of the selected redundant paths. The proposed approach can also be generalized to the case when network coding is allowed on internal nodes.

We are currently working to include in the same framework time dependence and exploit the time-diversity offered by packet interleaving [5][7]. The objective is a general framework where time-space-code diversity may be jointly analyzed for efficient transmission on computer networks.

References

[1] R. Ahlswede, N. Cai, S.-Y.R. Li, and R.W. Yeung, "Network Information Flow," *IEEE Transactions on Information Theory*, Vol. 46(4), pp. 1204–1216, July 2000.

[2] V.K. Goyal, and J. Kovacevic, "Generalized Multiple Description Coding with Correlating Transforms," *IEEE Transactions on Information Theory*, Vol. 47(6), pp. 2199–2224, Sept. 2001.

[3] S.-Y.R. Li, R.W. Yeung and N. Cai, "Linear Network Coding," *IEEE Transactions on Information Theory*, Vol. 49(2), pp. 371–381, July 2003.

[4] T. Nguyen, and A. Zakhor, "Path Diversity with Forward Error Correction (PDF) System for Packet Switched Networks," *Joint Conference of the IEEE Computer and Communications Societies* (INFOCOM 2003), Vol. 1, pp. 663–672, Mar. 2003.

[5] P. Salvo Rossi, G. Romano, F. Palmieri, and G. Iannello, "Interleaving for Packet Channels," *Conference on Computer Sciences and Systems* (CISS 2004), pp. 1560–1564, Mar. 2004.

[6] G. Romano, P. Salvo Rossi, and F. Palmieri, "Multiple Description Image Coder Using Correlating Transforms," *EURASIP European Signal Processing Conference* (EUSIPCO 2004), pp. 1321–1324, Sep. 2004.

[7] P. Salvo Rossi, F. Palmieri, G. Iannello, and A.P. Petropulu, "Packet Interleaving over Lossy Channels," *IEEE International Symposium on Signal Processing and Information Technology* (ISSPIT 2004), pp. 476–479, Dec. 2004.

[8] G. Romano, P. Salvo Rossi, and F. Palmieri, "Optimal Correlating Transforms for Erasure Channels," *IEEE Signal Processing Letters*, submitted to.